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Predictions From A U(2) Flavour Symmetry In Supersymmetric Theories. *

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Abstract

In a generic supersymmetric extension of the Standard Model, whether unified or not, a simple and well motivated U(2) symmetry, acting on the lightest two generations, completely solves the flavour changing problem and necessarily leads to a predictive texture for the Yukawa couplings.

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1. Introduction and Motivation

The accomplishment of the electroweak precision tests in the first phase of LEP [1], showing a remarkable agreement between the experimental results and the expectation of the Standard Model, has provided indirect evidence for the Higgs picture of the electroweak symmetry breaking. This brings the focus, more than ever, to the "fermion mass problem" of the SM: the inelegant description of fermion masses and mixings in terms of a number of arbitrary dimensionless Yukawa couplings of the fermions to the Higgs boson. These couplings show a strong hierarchical pattern, with only one coupling of order unity, largely dominating over the others, for which the SM provides no understanding.

Along an independent line of consideration, any evidence for the existence of the Higgs boson, as the one provided by LEP, strengthens the view that supersymmetry may be a relevant symmetry of nature, as realized, for example, in the Minimal Supersymmetric Standard Model. It is well known, on the other hand, that the "fermion mass problem", as defined above, appears in the MSSM precisely in the same way as in the SM itself. At the same time, however, it is also well known that the description of flavour in the MSSM has a special feature that distinguishes it from the SM [2, 3]. In the SM, the description of flavour in terms of Yukawa couplings provides a neat solution of the "flavour-changing" problem: the flavour changing neutral current processes are automatically suppressed to the required phenomenological level via the GIM mechanism [4]. On the contrary, for this to happen in the MSSM, appropriate universality assumptions for the soft supersymmetry breaking terms are also required as an independent input.

In supersymmetric theories the fermion mass and flavour-changing problems are different aspects of a single "flavour" problem. An interesting approach is to study flavour symmetries which simultaneously address both aspects [5]. The purpose of this paper is to show that a simple, well-motivated flavour symmetry, acting on the lightest two generations, completely solves the flavour-changing problem, and necessarily leads to a predictive texture for the Yukawa matrices.

There are many candidate flavour-symmetry groups G_f, each having several distinct symmetry breaking patterns [6]. In general, G_f must be contained in the full global symmetry group of the SM in the limit of vanishing Yukawa

couplings, $U(3)^5$, with each $U(3)$ acting in the 3-dimensional generation space and, independently, on one of the five irreducible representations under the "vertical" gauge group, (Q, u^c, d^c, L, e^c), which compose the usual 15-plet of matter fields per generation. Although our considerations, unless otherwise stated, do not depend on assuming a unified symmetry in the vertical direction, we nevertheless choose to restrict our attention to schemes which might be applicable in such a case too. In particular, if we consider the case of full unification of

$$\psi = (Q, u^c, d^c L, e^c) \tag{1}$$

into a single representation of the gauge group, as, e.g., in the case of $SO(10)$, we are lead to consider $U(3)$ as the maximal possible G_f. On the other hand, the large Yukawa coupling of the top quark, λ_t, represents a violent breaking of this family symmetry, which will reflect itself also in the sfermion spectrum, in fact both in the squarks and in the sleptons in the case of a unified theory [7]. For this reason, although the large λ_t might also result from the spontaneous breaking of the full $U(3)$ symmetry, we will mostly consider, in the following, a $U(2)$ family symmetry, under which the vertical multiplets ψ_i, $i = 1, 2, 3$ for the 3 families, transform as a $\underline{2} + \underline{1}$ representation [8].

An independent argument for considering only a $U(2)$ rather than the larger $U(3)$ symmetry is the following. Suppose, for definiteness, in the physical basis both for fermions and sfermions, that the mixing matrices in the gaugino-matter interactions are close to the standard CKM matrix. In such a case, a splitting between the masses of the first two generations of sfermions, of given charge, comparable to their mean mass leads to a serious flavour-changing problem, e.g., in $K^0 - \bar{K}^0$ mixing or in the rate for $\mu \to e\gamma$ or, if physical phases are present, in the CP violating ϵ parameter in K physics. By a related phenomenon, a large electric dipole moment for the electron and/or the neutron can also be generated. We will show how such problems can be taken care of by an appropriately broken $U(2)$ symmetry. On the contrary, a splitting between the third and the first two generations of sfermions is not necessarily a problem. It has actually been shown in minimal unified theories that the splitting produced in the sfermion masses as a consequence of the large top Yukawa coupling gives rise to very interesting signatures in lepton flavour violating processes [7] and in EDMs of the electron and the neutron [9, 7].

Hence we are led to consider $G_f = U(2)$, realizing that it is likely that this is a remnant of a strongly broken $U(3)$ flavour symmetry, the maximal flavour group for a unified theory.

2. The model

We first consider a set of general assumptions, which are:

i) The flavour group is $G_f = U(2)$, under which the three generations $\psi_i = \psi_a, \psi$, with $a = 1, 2$, transform as $\underline{2} + \underline{1}$.

ii) The Higgs field(s) H transform as some representation, reducible or irreducible, of the vertical gauge group G, but are pure G_f singlets.

iii) The flavour group G_f, which has rank 2, is broken by two vacuum expectation values: of a doublet ϕ^a and of a singlet, or, more precisely, a 2-index antisymmetric tensor ϕ^{ab}, such that, without loss of generality

$$\langle \phi^a \rangle = \begin{pmatrix} 0 \\ V \end{pmatrix}, \quad \left\langle \phi^{ab} \right\rangle = v\epsilon^{ab} \tag{2a}$$

For these vevs we assume the hierarchy $V \gg v$, so that

$$U(2) \xrightarrow{V} U(1) \xrightarrow{v} \text{nothing}. \tag{2b}$$

The most general G_f invariant superpotential relevant for generating fermion masses, W_Y, linear in H and bilinear in the matter fields, with non-renormalizable terms weighted by inverse powers of a mass scale $M \gg V \gg v$, is

$$W_Y = \psi\lambda_1 H\psi + \frac{\phi^a}{M}\psi\lambda_2 H\psi_a + \frac{\phi^{ab}}{M}\psi_a\lambda_3 H\psi_b + \frac{\phi^a\phi^b}{M^2}\psi_a\lambda_4 H\psi_b \tag{3}$$

At scale M we assume that the vertical gauge symmetry is reduced to the SM gauge group, so that, in general

$$\begin{aligned} \psi_i\lambda H\psi_j \; &= \lambda_U Q_i u_j^c h_2 + \lambda'_U u_i^c Q_j h_2 + \lambda_D Q_i d_j^c h_1 \\ &+ \lambda'_D d_i^c Q_j h_1 + \lambda_L L_i e_j^c h_1 + \lambda'_L e_i^c L_j h_1 \end{aligned} \tag{4}$$

For reasons that will become clear in the next Section, rather than considering the most general W_Y, we require that the non-renormalizable interactions

in the superpotential (3) be generated from a renormalizable superpotential by integrating out a heavy family, $\chi^a + \bar{\chi}_a$, vector-like under the vertical gauge group, transforming as a doublet under the flavour group. In full generality, such a superpotential is

$$W = \psi\lambda H\psi + \psi_a\lambda' H\chi^a + \phi^{ab}\psi_a\sigma\bar{\chi}_b + \chi^a M\bar{\chi}_a + \phi^a\psi\tau\bar{\chi}_a \tag{5}$$

Here, as in eq. 3, there is an implicit vertical structure for every term, which is left understood.

By integrating out the heavy fields $\chi^a + \bar{\chi}_a$, all terms of the superpotential (3) are reproduced, except the last one. In turn, this superpotential, after insertion of the vevs (2) , leads to the following texture of the Yukawa couplings for the Up quarks, the Down quarks and the charged Leptons

$$\lambda^{U,D,L} = \begin{pmatrix} O & d & O \\ -d & O & b \\ O & c & a \end{pmatrix}^{U,D,L} \tag{6}$$

where, setting $V/M = \epsilon$ and $v/M = \epsilon'$,

$$a = O(1) \;\; b, c = O(\epsilon) \;\; d = O(\epsilon'). \tag{7}$$

By an approximate diagonalization of these Yukawa couplings, taking into account the hierarchy in the mass eigenvalues, it is a simple matter to show that the CKM matrix takes the following form [10]

$$V_{CKM} = \begin{pmatrix} 1 & s_{12}^D + s_{12}^U e^{-i\phi} & s_{12}^U s_{23} \\ -s_{12}^U - s_{12}^D e^{-i\phi} & e^{-i\phi} & s_{23} \\ s_{12}^D s_{23} & -s_{23} & e^{i\phi} \end{pmatrix} \tag{8}$$

where

$$s_{12}^U = \sqrt{\frac{m_u}{m_c}}, \quad s_{12}^D = \sqrt{\frac{m_d}{m_s}}. \tag{9}$$

As a consequence, if we stick to the usual current algebra determination of the light quark masses[11], barring in particular $m_u = 0$ [12], this gives the relation

$$\left|\frac{V_{ub}}{V_{cb}}\right| = \sqrt{\frac{m_u}{m_c}} = 0.061 \pm 0.009, \tag{10a}$$

to be compared with the current world average [13]

$$\left|\frac{V_{ub}}{V_{cb}}\right|_{exp} = 0.08 \pm 0.02. \tag{10b}$$

Furthermore, it predicts

$$\left|\frac{V_{td}}{V_{ts}}\right| = \sqrt{\frac{m_d}{m_s}} = 0.226 \pm 0.009, \tag{11}$$

against a current range of 0.1 to 0.3, and, to account for the observed value of $|V_{us}| = 0.221 \pm 0.002$, it also predicts a large CP violating CKM phase, $\sin\phi \geq 0.9$.

How general is the form (8,9) that we have obtained for the CKM matrix? It is easy to see that the same form would have been obtained from the most general W_Y, since, in this case, the texture of the fermion mass matrices acquires also a non vanishing 22 entry, which does not affect eq. (8,9) [10]. Hence, a CKM matrix of the form (8,9), leading to predictions (10a, 11), results from an arbitrary theory at scale M, provided only that it possesses a $U(2)$ flavour symmetry broken below M by the two vevs of (2a). While this is a remarkably general result, the "renormalizable" model is of particular interest since it is the simplest viable U(2) invariant theory at scale M, and it possesses a very distinctive flavour and CP violating behaviour, induced by the gaugino-matter interactions.

The exchange of one heavy family, $\chi + \bar{\chi}$, singlet instead of doublet under G_f, would not have given rise to a realistic texture. On the contrary, as we are also going to show, a set of heavy matter fields transforming under G_f as $\underline{2} + \underline{1}$, $\chi + \bar{\chi} + \chi^a + \bar{\chi}_a$, maintains the same properties of the pure heavy doublet model provided the coupling $\psi_a \phi^a \bar{\chi}$ in the superpotential is forbidden by an appropriate extension of the flavour symmetry.

3. Sfermion masses

As anticipated, the $U(2)$ flavour symmetry not only constrains the form of the Yukawa couplings, but also the amount of non-degeneracy between the sfermions of the first and second generation. In turn, this is the key quantity which affects the "flavour changing problem" in a supersymmetric theory.

Supersymmetry breaking is assumed to occur as in supergravity [14], with no universality-type constraint on the scalar masses or on the analytic terms, both characterized in the usual way by a scale $m \ll M$. In the $U(2)$ invariant limit, the scalar mass squared matrices for each charge are diagonal with the first two entries degenerate. Consider the general $U(2)$ invariant theory, based on the assumptions i) - iii), with F terms given by (3). Since ϕ appears only in the non-renormalizable terms, as $M \to \infty$ the effects of $U(2)$ breaking decouple. The same holds true for the D terms where all $U(2)$ breaking is again suppressed by powers of M, for example, as in the soft operator: $m^2 \psi^* \psi_a \phi^a / M$. Hence the deviation of the scalar mass matrix from the $U(2)$ invariant form is described by entries involving powers of ϵ and ϵ', the same small parameters that generated the hierarchies in the fermion mass matrices. In particular, because $U(2)$ is kept as a good symmetry beneath the scale M of any new heavy particles having renormalizable interactions with the light matter, the potentially dangerous radiative corrections to universality [15] are suppressed by powers of ϵ and ϵ'. From a general operator analysis one finds that the sfermions of the first and second generations are split by a relative amount of order ϵ^2, so that the typical supersymmetry breaking scalar masses of the i-th generation, m_{Si}, are related to the i-th generation fermion masses, m_{Fi}, by

$$\frac{m_{S1}^2 - m_{S2}^2}{m_{S1}^2 + m_{S2}^2} = O\left(\frac{m_{F2}}{m_{F3}}\right) \tag{12}$$

leading to a problematic contribution to the ϵ parameter of kaon physics [8].

In the "renormalizable" model of (5), the $U(2)$ flavour symmetry is broken by interactions of ϕ^a and ϕ^{ab} coupling light and heavy generations. Any such mass mixing effect generates at tree level both fermion mass hierarchies [16] and deviations from the flavour symmetric form of the scalar mass matrices [17]. The latter effect is in general a powerful constraint on supersymmetric theories of fermion masses which use the Froggatt-Nielsen mechanism, and is more dangerous than the radiative effects of [15]. However, in the present case, with the exchange of only a heavy $U(2)$ doublet generation, a non-degeneracy between the scalar masses of the first two generations is induced only at order $\epsilon^2 \epsilon'^2$, or

$$\frac{m_{S1}^2 - m_{S2}^2}{m_{S1}^2 + m_{S2}^2} = O\left(\frac{m_{F1} m_{F2}^2}{m_{F3}^3}\right) \tag{13}$$

making any effect in flavour and/or CP violations due to the splitting between m_{S1} and m_{S2} completely negligible. This arises because ϵ is the only SU(2) breaking parameter and because, in the limit of vanishing ϵ', corresponding to massless fermions of the first generation, the exact composition of the heavy states, as determined by the superpotential of eq. (5) does not contain the light doublets ψ_a at all. This means that, as $\epsilon' \rightarrow 0$, in the most general supersymmetry breaking potential (apart from terms linear in H)

$$\begin{aligned} V &= m_1^2 \left|\psi_a\right|^2 + m_2^2 \left|\chi^a\right|^2 + m_3^2 \left|\psi\right|^2 + m_4^2 \left|\bar{\chi}_a\right|^2 \\ &+ \phi^{ab}\psi_a A_1 \lambda_1 \bar{\chi}_b + \chi^b A_2 M \bar{\chi}_b + \phi^b \psi A_3 \sigma \bar{\chi}_b \end{aligned} \tag{14}$$

the scalar components of the light first two generations do not feel at all the breaking of the $U(2)$ symmetry and therefore remain exactly degenerate. An explicit calculation, including the ϵ'-terms, shows that the corrections to exact degeneracy are of order $\epsilon^2\epsilon'^2$. Precisely this same argument can be repeated in the case of heavy matter states consisting of a $\underline{2} + \underline{1}$ representation under G_f. In this case, the decoupling of the light doublets as ϵ' goes to zero requires that the coupling $\psi_a \phi^a \bar{\chi}$ be forbidden by an appropriate extension of the flavour symmetry.

4. Flavour and CP violations

As particularly important characteristic observables, we consider the rate for the decay $\mu \rightarrow e\gamma$, the CP violating parameter ϵ in K physics and the electric dipole moments for the electron and/or the neutron. For given values of the mixing angles and particle masses, the various contributions to these observables from one loop supersymmetric particle exchanges have been computed, e.g., in reference [7].

As mentioned, in the "renormalizable" model, no sizeable contribution to these observables is expected from the exchanges of the highly degenerate sfermions of the first and second generation. On the other hand, a calculation of the contribution from the exchange of the third generation sfermions would require knowing the splitting of their mass with respect to that of the first two generations, which is not determined by pure symmetry arguments. Still the pattern of masses characteristic of this model allows several precise considerations to be made.

Let us work in the superfield basis where the sfermion squared mass matrices are diagonal. In this basis, the fermion masses are diagonalized as usual by

$$M^{U,D,L} = V_\ell^{U,D,L} M_{diag}^{U,D,L} V_r^{\dagger U,D,L} \tag{15}$$

where the matrices V_ℓ and V_r define the mixing matrices in the gaugino-matter interaction vertices. Taking into account the texture of (6), it is immediate to see that V_ℓ and V_r have the following approximate form

$$V_{\ell,r}^{U,D,L} = \begin{pmatrix} 1 & s_{12}^{\ell,r} & O \\ -s_{12}^{\ell,r} & 1 & s_{23}^{\ell,r} \\ s_{12}^{\ell,r} s_{23}^{\ell,r} & -s_{23}^{\ell,r} & 1 \end{pmatrix}^{U,D,L} \tag{16}$$

where

$$(s_{23}^\ell s_{23}^r)^{U,D,L} = \left(\frac{m_2}{m_3}\right)^{U,D,L} \quad - s_{12}^{\ell U,D,L} = s_{12}^{r U,D,L} = \sqrt{\frac{m_1}{m_2}}^{U,D,L} \tag{17}$$

and all phases have been eliminated by phase redefinitions of the superfields u, u^c, d, d^c, e, e^c. That this is possible at all is a non trivial property of the texture (6). The implication of this for CP violation is discussed below.

In general, the theory contains two sources of flavour violations: the mixing matrices V_ℓ and V_r and the A-terms linear in H, whose general form, before integrating out the heavy fields, $\chi^a + \bar{\chi}_a$ is

$$\psi A \lambda H \psi + \psi_a A' \lambda' H \chi^a \tag{18}$$

In turn, the $\mu \to e\gamma$ decay amplitude receives a contribution from the mixing matrices in the gaugino-higgsino interactions and another from the A-terms. The first one is proportional to $V_{\ell_{31}}^L m_\tau V_{r_{32}}^L$, or $V_{r_{31}}^L m_\tau V_{\ell_{31}}^L$, both determined, from eq.(17), to be equal to $\sqrt{m_e m_\mu}$, which is a factor of ten bigger than $V_{td}^{CKM} m_\tau V_{ts}^{CKM}$, as arises in minimal $SO(10)$ [7]. Leaving aside the A-term contributions, which contain one unknown parameter for any independent Yukawa coupling, the $\mu \to e\gamma$ decay rate, for $m_{gaugino} = m_{sleptons} = m$, is estimated as

$$BR(\mu \to e\gamma) = O(10^{-10}) \left(\frac{300GeV}{m}\right)^4 \left(\frac{\Delta_\ell}{m^2}\right)^4 \tag{19}$$

where $\Delta_\ell = m_{\tilde{\tau}}^2 - m_{\tilde{\mu}}^2$ is the splitting between the stau and smuon masses.

Let us turn now to CP violation and consider first the leptons. As aforementioned, all phases can be eliminated from the mass matrix by redefining the fields L and e^c. We assume that the only source of CP violation is in the Yukawa couplings, so that the A parameters, the gaugino masses and the μ-term are all real. This means that the same redefinition of the L and e^c superfields which makes the mass matrix real, also makes real the A-terms arising from eq. (18). Under the stated assumptions, there is no CP violation in the lepton sector: in particular there is no one loop contribution from gaugino-slepton exchanges to the electric dipole moment of the electron.

At variance with the lepton case, the phases of the quark mass matrices can only be eliminated by independent redefinitions of the u and d fields, which of course explains why CP is violated, as shown by the presence of the physical phase in the CKM matrix, eq. (8). Nevertheless CP violation is more "screened" than in the generic case.

Let us consider first the dipole moments, electric or chromoelectric, (EDM), of the quarks. In all the one loop diagrams with internal squarks possibly contributing to the EDM of the up or the down quarks, only the mass terms or the A-terms involving the u^c or the d^c are respectively relevant. It is therefore possible, by redefining the superfields Q and u^c, or Q and d^c, to rotate away all phases both from the mass terms and from the supersymmetric one loop dipole moments for the up and the down quarks. As in the lepton case, there is no one loop EDM for the quarks too.

By working in the basis where the d-quark mass has been made real, it is also clear that the supersymmetric box-diagram contribution to the $\Delta S = 2$ left-right effective Hamiltonian operator $(\bar{s}_L \gamma_\mu d_L)(\bar{s}_R \gamma_\mu d_R)$ has a real coefficient. In this same basis, however, the conventional $\Delta S = 1$ operator $(\bar{s}_L \gamma_\mu d_L)(\bar{u}_L \gamma_\mu u_L)$ has a complex coefficient from the CKM matrix element V_{us} of eq. (8), which induces an ϵ parameter. Taking into account that the gluino exchange contribution to the $\Delta S = 2$ operator is proportional to

$$\left(V^D_{\ell_{31}} V^D_{r_{32}} \right)^2 = \frac{m_d m_s}{m_b^2}, \tag{20}$$

a factor of about one hundred times bigger than $(V^{CKM}_{td} V^{CKM}_{ts})^2$, which would

arise in minimal $SO(10)$ [7], this leads to

$$\epsilon = O(10^{-2}) \left(\frac{1TeV}{m}\right)^2 \left(\frac{\Delta_q}{m^2}\right)^2 \sin\phi \tag{21}$$

for $m_{gluino} = m_{squark} = m$, where $\Delta_q = m^2_{\tilde{b}} - m^2_{\tilde{s}}$ is the splitting between the relevant squark masses. Recall that the observed value of V_{us} suggests a large value for the CP violating phase, or $|\sin\phi| \approx 1$.

5. Conclusions

In this paper we have described a supersymmetric model which solves the "flavour-changing problem" by virtue of a spontaneously broken, non-Abelian flavour symmetry and, at the same time, forces an interesting texture of fermion masses which leads to some predictions for the CKM parameters. The masses of the sfermions of the first two generations are highly degenerate, resulting in a strong suppression of the related contributions to the flavour and/or CP violating observables.

A detailed prediction of all such observables would require knowing the splitting between the third and the first two generations of squarks and sleptons, which is not fixed by pure symmetry considerations. In any case, under the stated assumptions, we can say that we do not expect sizeable EDMs for the quarks or the electron. On the contrary, both the $\mu \rightarrow e\gamma$ decay rate and the ϵ parameter in K physics receive significant contributions from supersymmetric one loop diagrams if the splitting between the third and the first two generations of squarks and sleptons, $\Delta_{q,l}$, is indeed sizeable. In a unified theory, the maximal flavour symmetry, $U(3)$, is necessarily strongly broken to $U(2)$ by the large top quark Yukawa coupling, and we know of no mechanism which is able to protect a small scalar mass splitting $\Delta_{q,l}$. In establishing the relative importance of $\mu \rightarrow e\gamma$ and ϵ, the family independent effect of the gluino on the squark masses may play a role [7].

In previous papers, two of us have shown that [7], in a unified theory, the large top Yukawa coupling is indeed a source of significant splitting between the masses of the third generation sleptons and squarks with respect to the first and second generation sfermions of the same charge. In computing the induced flavour and CP violations, the effect of a possible splitting between the first and

second generation sfermions was ignored there. As pointed out in the introduction, such an effect, if indeed existing, would have been much bigger. Hence a possible objection to that work is that the effects arising from the splittings $\Delta_{q,l}$ should not be trusted in theories where potentially disastrous, larger effects have not been considered. Although the considerations in the present paper have not been specifically addressed to the case of a unified theory, they can nevertheless be extended to it. In this way, we think that our previous conclusions [7] about the relevance of flavour signals in supersymmetric unification are reinforced.

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